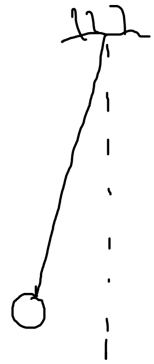
Free Oscillations

Dr K M Hock

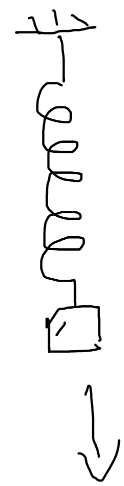
e.g.



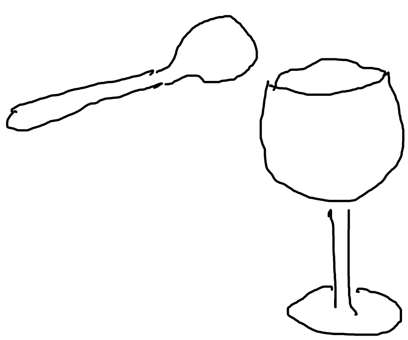
Just pull,
let go...

e.g.

Again pull,
let go...



e.g



Tap with
a spoon...

then they oscillate/vibrate on their
own – no outside force needed.

Oscillator Motion

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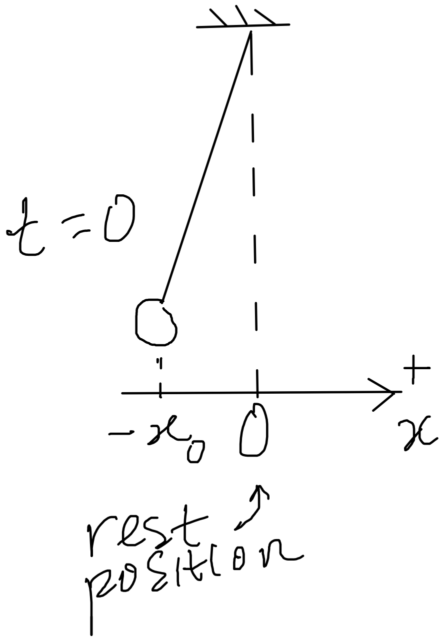
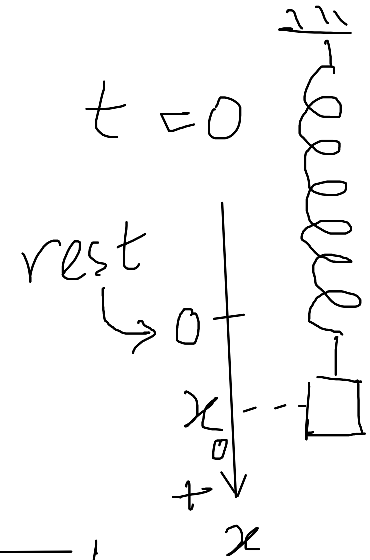
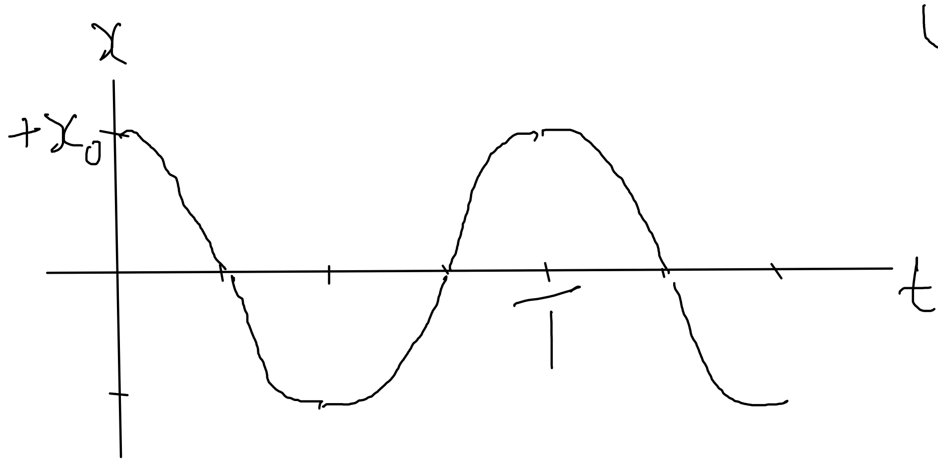


$$x = x_0 \sin \omega \left(t + \frac{T}{4}\right)$$



$$x = x_0 \sin \omega \left(t - \frac{T}{4}\right)$$



- They look like sine graph.
- But can start at any time.
- And 1 cycle can have any time T.
- $x = x_0 \sin \omega t$ just 1 example.

understand and use the terms amplitude, period, frequency, angular frequency and phase difference and express the period in terms of both frequency and angular frequency

Describing Oscillation

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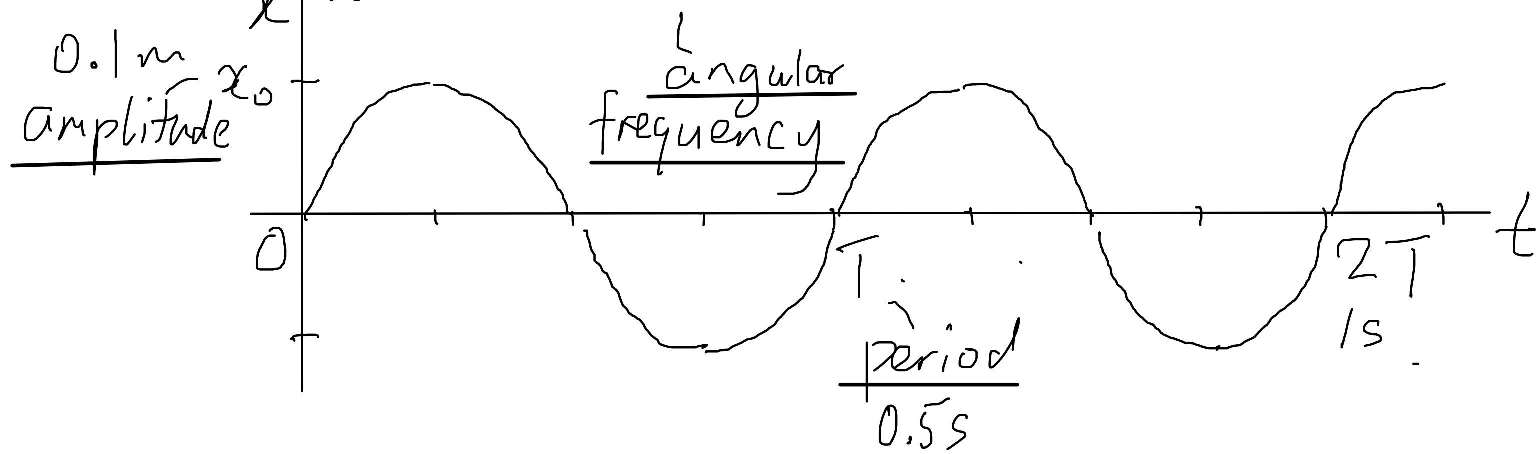
$x = x_0 \sin \omega t$



frequency = no. of cycles per unit time

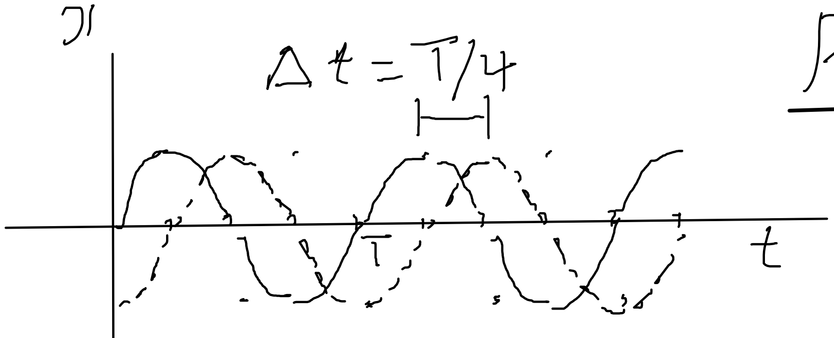
e.g. $f = \dfrac{1 \text{ cycle}}{0.5 s} = 2$ cycles/s.

So $\boxed{f = \dfrac{1}{T}}$ (Hz)

Note: $x = x_0 \sin \omega t \rightarrow 0 = \cancel{x_0} \sin \omega T$

$\therefore \underbrace{\omega T}_{1 \text{ cycle}} = 2\pi$, $\boxed{\omega = \dfrac{2\pi}{T} = 2\pi f}$

ω = angular frequency (rad/s)



Phase difference

$\phi = \dfrac{\Delta t}{T} \times 2\pi$ (rad) recognise and use the equation a = – ω^2 x as the defining equation of simple harmonic motion

Simple Harmonic Motion

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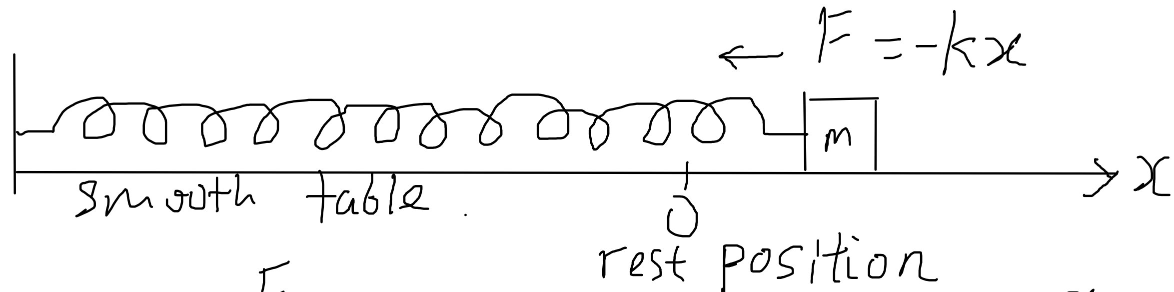
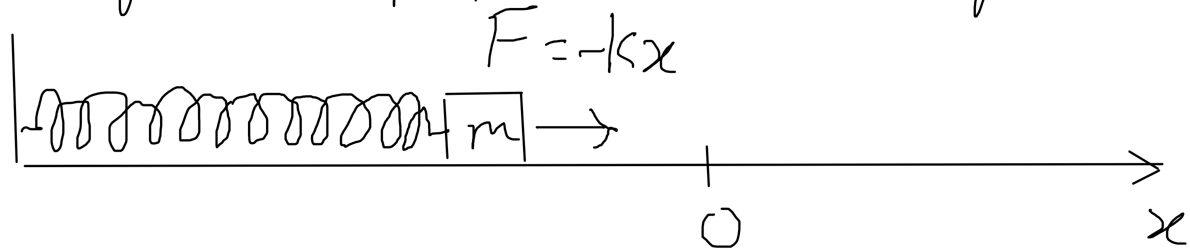
e.g.



Restoring force (F) proportional to displacement (x).



F has -ve sign ∴ opposite direction to x.

Since $F = ma$, so also a proportional to x.

$$\therefore \quad a = -\text{constant} \times x.$$

Using calculus, can find a solution:

$$x = x_0 \sin \omega t$$

where ω^2 = constant above

$$\therefore \quad \boxed{a = -\omega^2 x}$$

Define SHM: motion where restoring force on body proportional to displacement from fixed point, but opposite to " .

Simple Harmonic Solution 2

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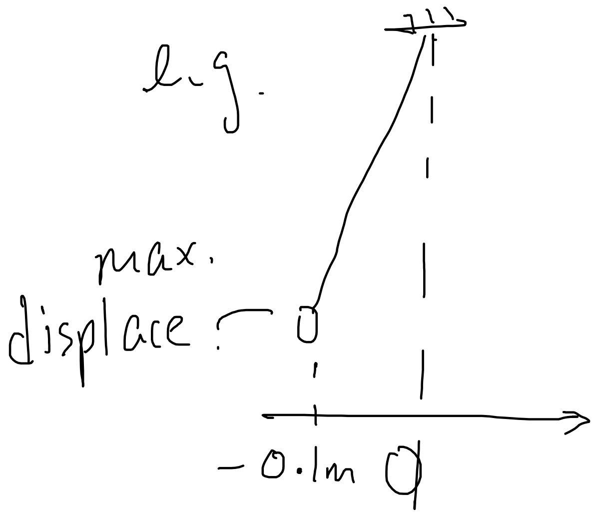
e.g.



Given that acceleration

$$a = -16x.$$

(a) Find the period.

(b) Find a solution x.

(c) Sketch graph $x-t$.

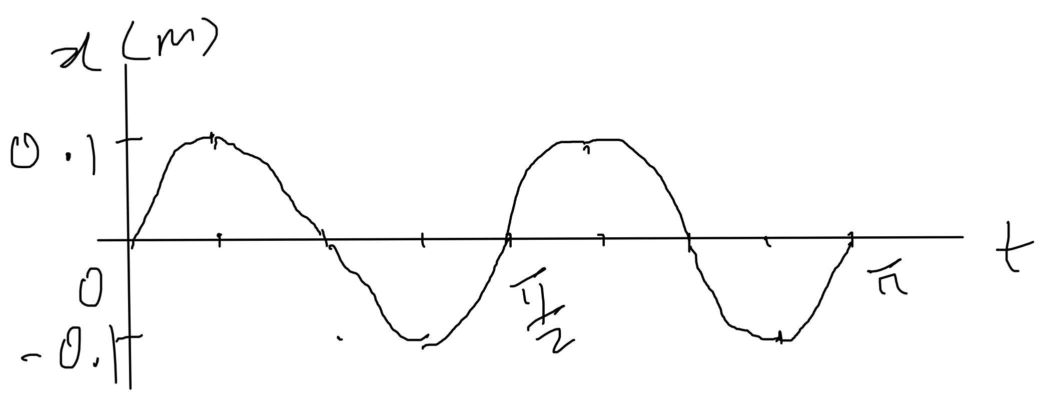
(a) $a = -\omega^2 x$ $\qquad \omega^2 = 16$, $\omega = 4$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{4} = \frac{\pi}{2}\ \text{s}.$$

(b) $x = x_0 \sin \omega t$

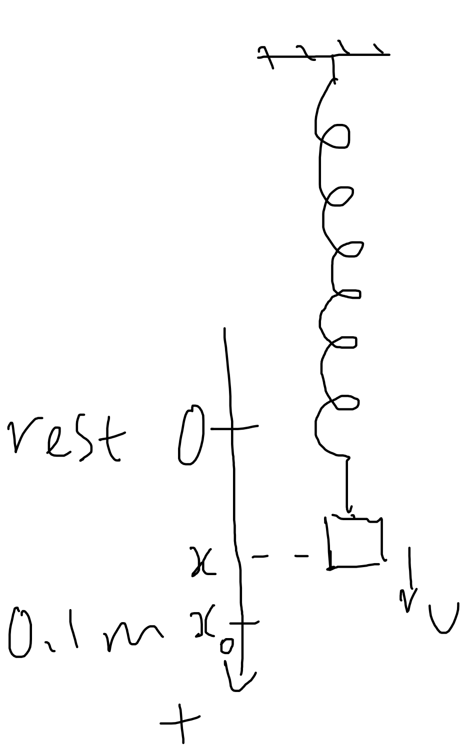
$$x = 0.1 \sin \frac{\pi}{2} t$$

(c)



recognise and use v = v_0 cos ω t and ω = ±(x_0^2 - x^2)^(1/2)

Simple Harmonic Solution 3 Dr K M Hock



If $x = x_0 \sin \omega t$,

then velocity

$$v = v_0 \cos \omega t$$

$$v = \pm \omega \sqrt{x_0^2 - x^2}.$$

e.g. given $T = 1$ s.

(i) Find v when $x = 0.05$ m.

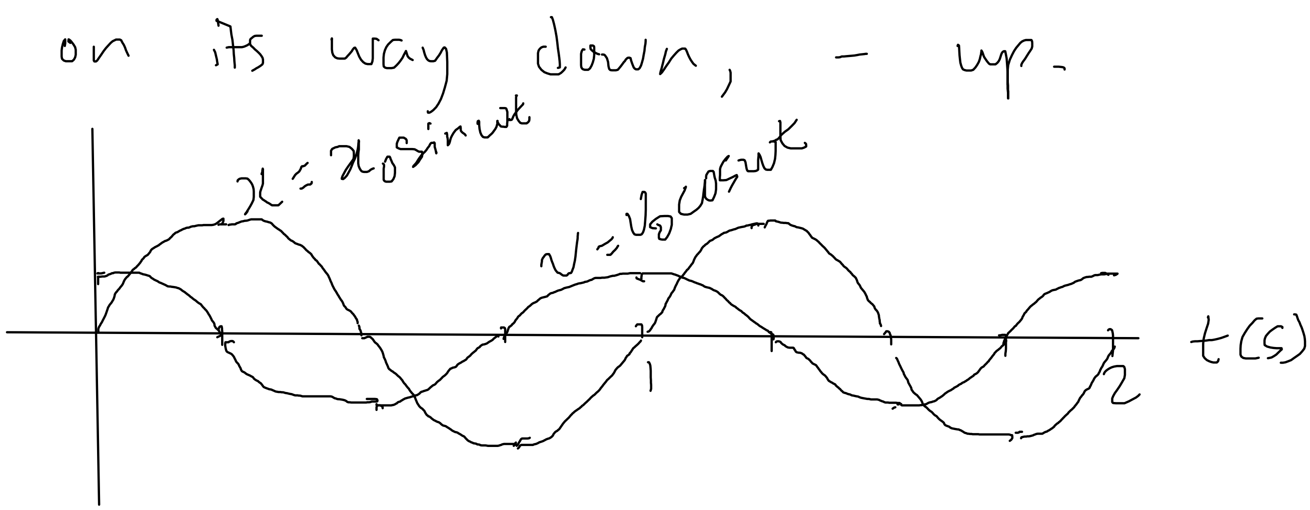
(ii) Sketch x-t, v-t on same graph.

(i) $\omega = \frac{2\pi}{T} = \frac{2\pi}{1} = 2\pi$ rad/s.

$$V = \pm 2\pi \sqrt{0.1^2 - 0.05^2} = \pm 0.5441 \text{ m/s}$$

+ on its way down, − up.

(ii)



SHM Graphs

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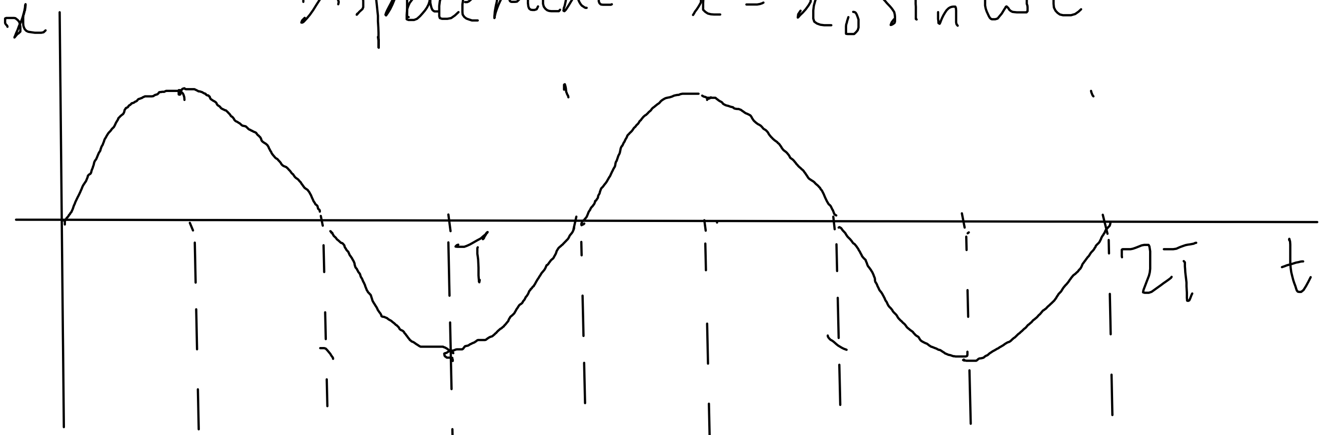
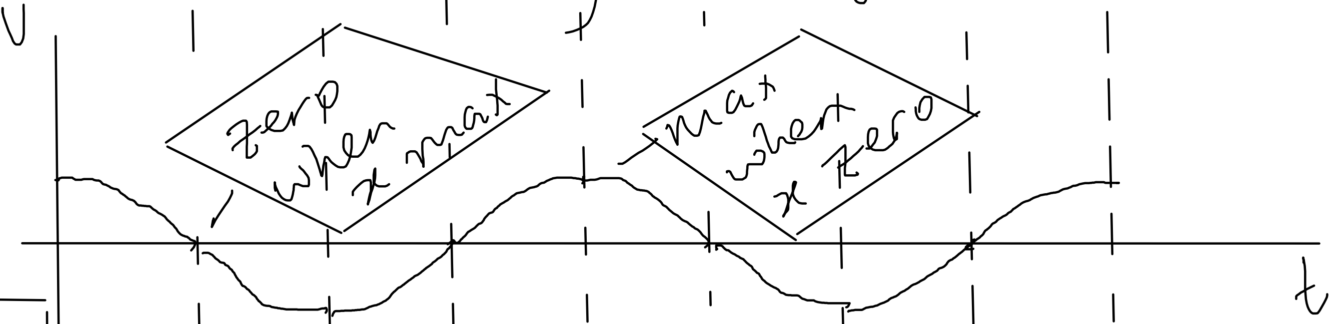
Displacement $x = x_0 \sin \omega t$

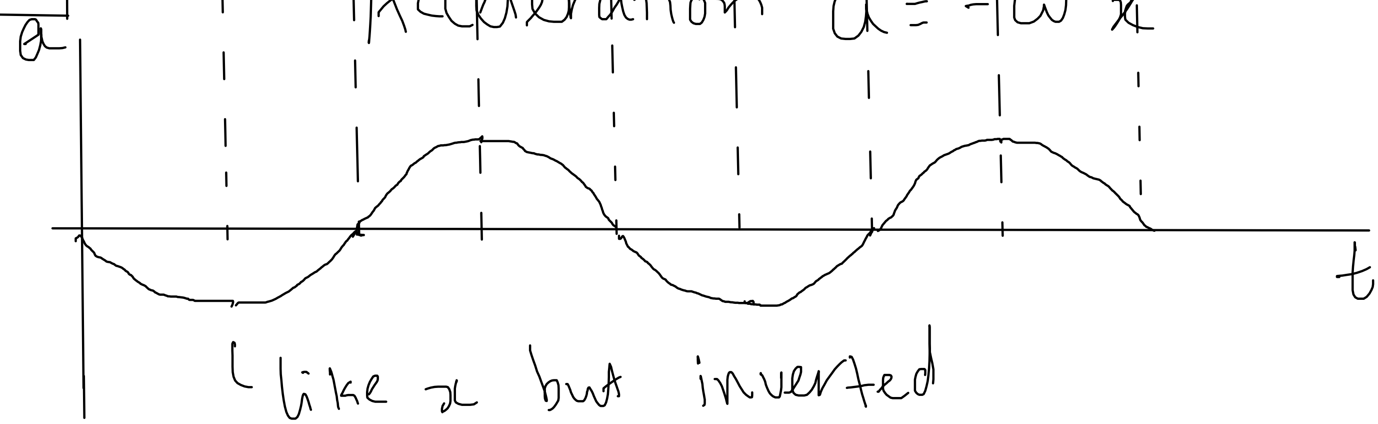


Velocity $v = v_0 \cos \omega t$



* when v is max a is ZERO!

Acceleration $a = -\omega^2 x$



like x but inverted
- because restoring force opp- displacement

describe the interchange between kinetic and potential energy during simple harmonic motion

Kinetic, Potential Energies

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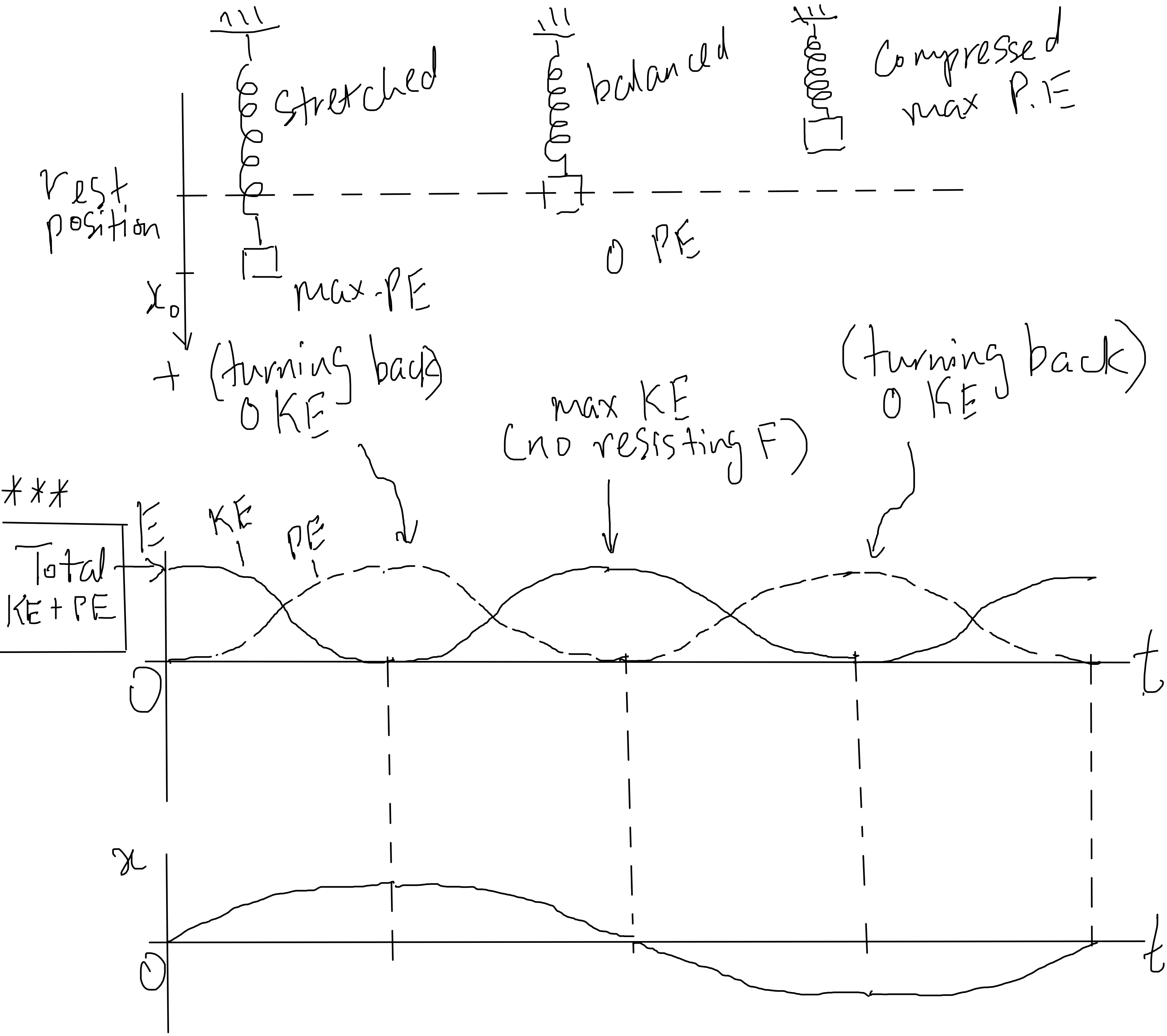

describe practical examples of damped oscillations with particular reference to the effects of the degree of damping and the importance of critical damping in cases such as a car suspension system

Damped Oscillations

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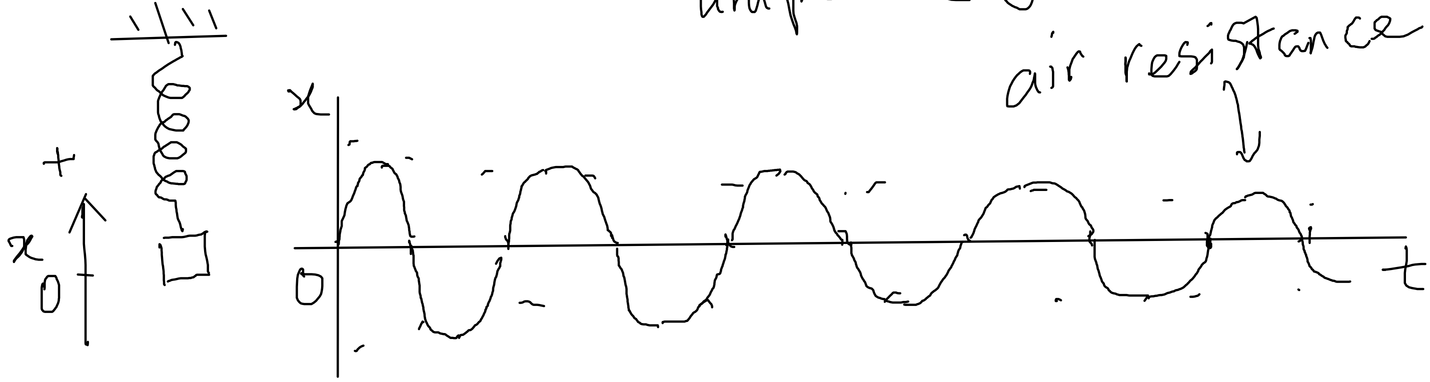
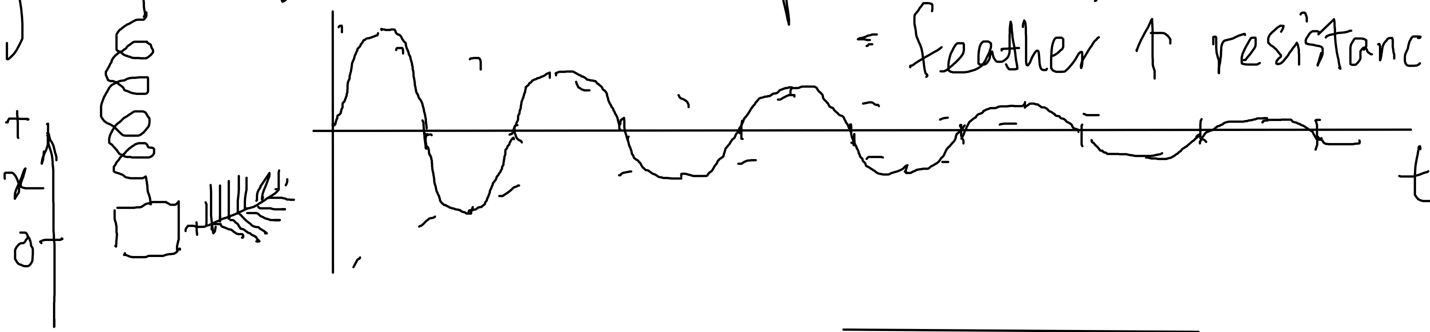
amplitude ↓ - air resistance



e.g. -

amplitude ↓ faster - feather ↑ resistance



Resistance - called $\boxed{\text{Damping}}$.

Amplitude ↓ - $\boxed{\text{Damped Oscillation}}$

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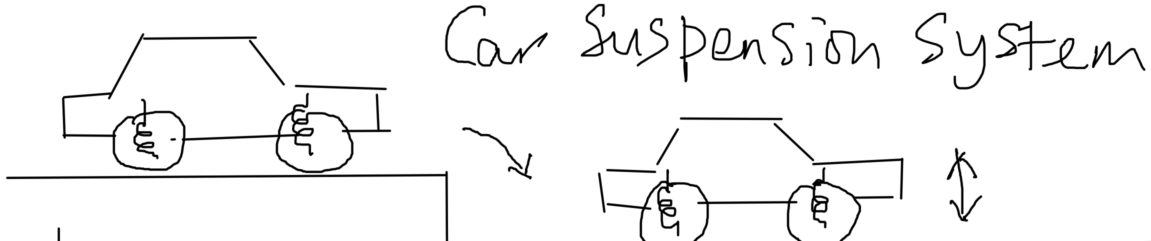
e.g. Car Suspension System



Can be bumpy ...

Adjust damping ...

$\boxed{\text{Critical damping}}$ → until oscillation <u>just</u> stops.

Forced Oscillation

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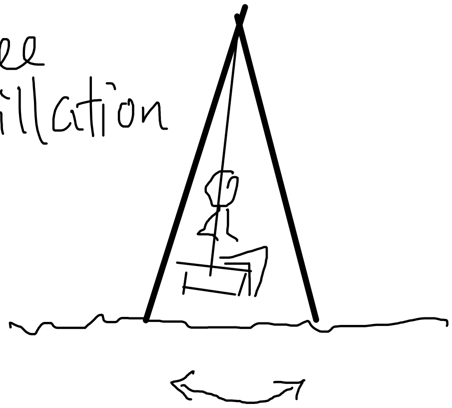
free oscillation



(natural) frequency f_0

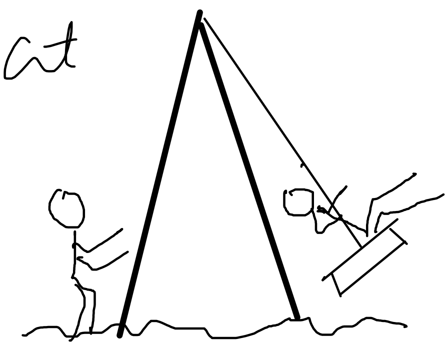
e.g 1 Hz.

forced oscillation

frequency f

can be different

But if pushed at natural freq:



Amplitude can get quite big

→ $\boxed{\text{Resonance}}$

e.g. of forced oscillations:

loudspeaker, radio transmitter, violin, electric saw, ...

describe graphically how the amplitude of a forced oscillation changes with frequency near to the natural frequency of the system, and understand qualitatively the factors which determine the frequency response and sharpness of the resonance

Resonance

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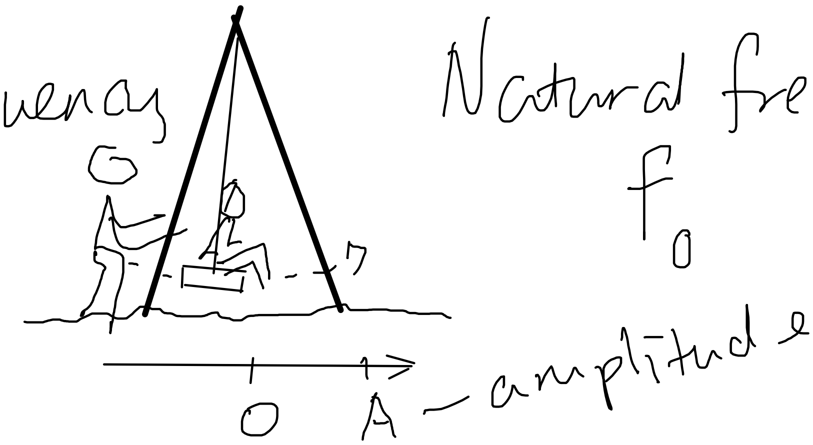
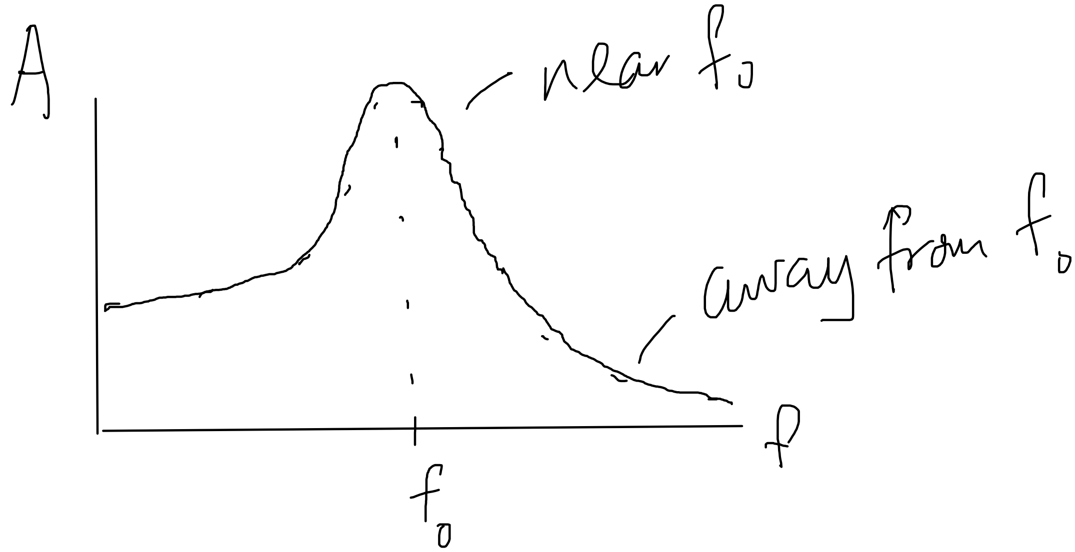
Forced frequency f

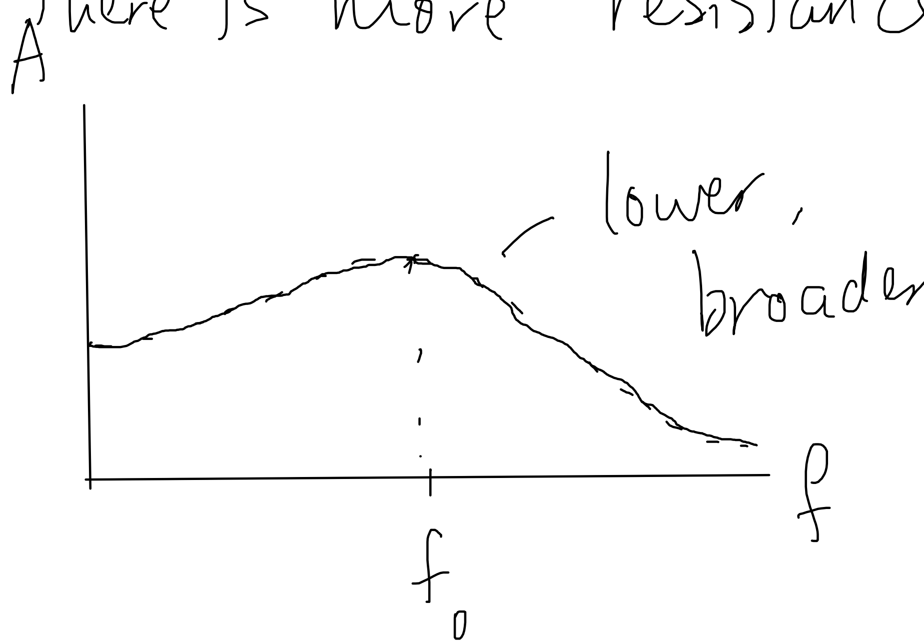
Natural frequency f_0



Resonance graph



But if there is more resistance (friction, wind, ...)



show an appreciation that there are some circumstances in which resonance is useful and other circumstances in which resonance should be avoided.

Good, Bad Resonance

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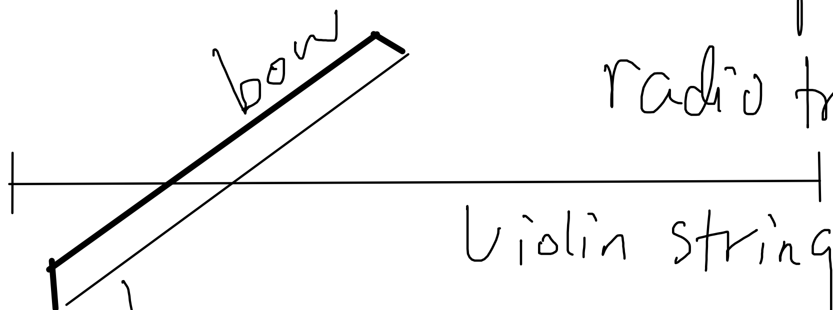
Good Resonance : musical instrument, loud speaker, radio transmitter, ...



- rough surface rubs string at many frequencies
- string oscillation increases if rubbing has natural frequency

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Bad resonance :
- Tacoma bridge
- voice break glass
- bad car suspension, ...

Singer